Evolutionary Game Theory

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Postgridiots, 21 Oct 2014

Abstract

I review some basic concepts from Evolutionary Game Theory (EGT), a general mathematical framework for thinking about the competition and propagation of features or traits in arbitrary populations. While the origins of EGT lie in evolutionary biology, particularly in modelling individual contests in environments with limited resources, the framework has potential applications in fields as diverse as sociology, economics and linguistics, and adoption of the framework need not imply any sort of "biologism", as I will argue. The framework does, however, hinge crucially on the notion of fitness or utility, which may be cause for some worry. I illustrate its workings with the replicator–mutator equation, a simpl(e|istic) way of modelling the dynamics of something like competing parameterizations of UG in syntax, and note that interesting things are happening in semantics and pragmatics as well, though not many people are aware of them and a resistance to exploring idealized mathematical models exists within the linguistic community.

1   "Evolutionary game theory is a way of thinking about evolution at the phenotypic level when the fitnesses of particular phenotypes depend on their frequencies in the population." (Maynard Smith, 1982, 1.)

1.1   **Optimization** problems have a long history in biology: how does an individual organism, or a species as a whole, adapt to a given environment? If, let us say, an organism's genome has a **fitness landscape** so that having a particular sequence of alleles gives an optimal phenotype, how is that optimal sequence found (mathematically)? and: How does the species find it (naturally)? Most modern answers would be mathematical explications of the basic Darwinian premise, evolution by natural selection.

1.2   Such a framework is reasonable when the success of an individual does not depend on what other individuals are doing. When such inter-individual effects are considered, simple optimization is not enough, and we need a framework for thinking about things like contests and co-operation between individuals.

1.3   Example (Maynard Smith, 1982, 1): evolution of wing form in a bird species is an adaptation to atmospheric conditions and certain physical facts such as drag and lift forces; evolution of dispersal behaviour in the same bird is another kind of development which crucially depends on how conspecifics are behaving.

1.4   It is the latter kind of situation that Evolutionary Game Theory (EGT) attempts to model, combining mathematical machinery from the theory of dy-

namical systems and Game Theory, the latter of which was first applied in the context of economics starting in the 1940s. Research in EGT has been active ever since the 1970s and especially since the publication of Maynard Smith's now classic *Evolution and the Theory of Games* (Maynard Smith, 1982), the first book-length synthesis. As important biological precursors, we can mention Lotka's and Volterra's research into predator–prey interaction in the early 20th century, pioneering studies of ecological dynamics which can be subsumed under the modern treatment of EGT.

2   "…any behavioural phenotype, in fact any phenotype at all, may be viewed as a strategy, as long as we wish to analyse it by game theoretical means…" (Hofbauer & Sigmund, 1988, 116.)

2.1   The dynamics of a population can be conceived as a **game**.

2.2   In such a game individual members of the population constantly come into contact with each other. The individuals employ one or more **strategies** from a set of possible strategies. When two or more strategies meet, a **payoff** (positive, negative, or indifferent) results for each of the individuals concerned. In a typical Darwinian case, we would factor this payoff into the **fitness** of each individual, so that negative payoffs decrease and positive payoffs increase the fitness of the individual or of the strategy, depending on your stand on the level-of-selection debate.

2.2.1   The dynamics of such a population then depends on at least the following things: (i) the manner in which individuals come into contact with each other, (ii) how payoffs are calculated, (iii) what the frequency of each possible strategy is in the population at a given point in time.

2.3   Example: **Hawks and Doves**. Imagine a state of contest between two animals (or species) over a resource of which there is a bounded amount (e.g. food). Let this resource have a value V, which we interpret as the amount by which the fitness of an individual increases if it manages to secure the resource. Assume that the strategy set has two elements:

(1) Hawk: if in a contest, always escalate and continue the contest until injured or opponent retreats

(2) Dove: if in a contest, engage in "display" behaviour but do not escalate; retreat at once if opponent escalates, abandoning the resource

Let's assume that injury decreases the fitness of an individual by an amount C. We can now write a **payoff matrix** for this particular game:

	Hawk	Dove
Hawk	$(V-C)/2$	V
Dove	0	$V/2$

or

$$B = [b_{ij}] = \begin{pmatrix} (V-C)/2 & V \\ 0 & V/2 \end{pmatrix}. \tag{1}$$

How to read this? Cell b_{11} gives your payoff if you're a Hawk and find yourself in a contest with another Hawk: in such a case, we assume that each Hawk has an equal probability of securing the resource ($V/2$) and being injured ($C/2$).

Cell b_{12} gives your payoff if you're a Hawk against a Dove: in such a case, the Dove immediately retreats and your payoff is V. And so on. In a contest between Doves we assume that one of them eventually gets bored of the display choreography and retreats.

2.4   The original motivation for studying games like Hawk–Dove was to understand how something like "limited war" behaviours, or "conventional fights" (feather fluffing, pushing matches, etc.) could be sustained by natural selection: is it possible, even in principle, for a strategy like Dove which always displays but never escalates, to be stable if natural selection operates on individuals rather than species? (I.e. could a limited war strategy be adaptive for an individual as well as for a species?)

2.4.1   We would then like to ask questions like: can Dove strategists be sustained in a population with a nonzero density of Hawk strategists (are mixed populations stable)? Can a Dove invade a Hawk population, or vice versa? What's the fate of a **mixed strategist**, someone who plays Dove some of the time and Hawk the remaining time? More generally: what are the equilibria of a system like Hawk–Dove, and are they stable or unstable? If we start the system from some initial condition and let time tend to infinity, to what sort of state will the system tend?

2.5   An **evolutionarily stable strategy** (ESS) is a strategy with the following property: if all the members of a population were to adopt it, no other (mutant) strategy could invade the population.

2.5.1   Now consider a two-strategy game such as Hawk–Dove. The simplest way of defining the **fitness** of each strategy is to take (1) the payoff from playing this strategy against itself and (2) the payoff from playing this strategy against the other strategy, to (3) weigh these with the densities of both strategies in the overall proportion, and (4) to take the sum. So, we define the fitnesses as

$$\begin{cases} f_1 = x_1 b_{11} + x_2 b_{12} \\ f_2 = x_1 b_{21} + x_2 b_{22} \end{cases}, \tag{2}$$

where x_1 denotes the relative frequency (proportion, density) of Hawks and x_2 that of Doves in the population.

2.5.2   Suppose now that x_2 is very small, so that the population consists mostly of players of strategy 1. For strategy 1 to be an ESS we want that $f_1 > f_2$. This happens iff (E1) $b_{11} \geq b_{21}$ and (E2) if $b_{11} = b_{21}$, then $b_{12} > b_{22}$.

2.5.2.0.1   More formally, let $x_2 = \epsilon$ for any small $\epsilon > 0$. Then the requirement $f_1 > f_2$ becomes

$$(1 - \epsilon)b_{11} + \epsilon b_{12} > (1 - \epsilon)b_{21} + \epsilon b_{22}, \tag{3}$$

which it is also possible to write as

$$(1 - \epsilon)(b_{11} - b_{21}) + \epsilon(b_{12} - b_{22}) > 0. \tag{4}$$

Now, (4) holds if and only if (E1) $b_{11} \geq b_{21}$ and (E2) if $b_{11} = b_{21}$, then $b_{12} > b_{22}$. The first requirement follows from the fact that (4) must hold for arbitrarily small ϵ.

2.5.2.0.2 In plain terms, the two conditions mean: for strategy 1 to be an ESS, (E1) it has to be the best response to itself, and (E2) if some other strategy is as good a response, strategy 1 must be a better response to that strategy than is that strategy to itself.

2.5.2.1 This is the formal definition of an ESS: a strategy is an ESS iff both (E1) and (E2) hold. It generalizes to games with more than two strategies.

2.5.3 With these notions in hand, it is easy to make the following observations concerning Hawk–Dove:

2.5.3.1 Dove is an ESS if and only if $0 = V < C$.

2.5.3.2 Hawk is an ESS if and only if $0 \leq C \leq V$.

2.5.3.3 If $0 < V < C$, there is no ESS.

2.6 An **evolutionarily stable state** is a state of the population which satisfies the following criteria: (S1) it is an equilibrium, i.e. if the system is in this state, the frequencies of the different strategies will not change; (S2) it is stable, i.e. if the system is perturbed a bit from this equilibrium, it will return to the equilibrium.

2.6.1 This is a more general concept than that of an ESS: in an evolutionarily stable state, it may be the case that no pure strategy is stable, but a mix of strategies is.

2.6.2 In fact, it turns out that such a stable strategy mix exists for Hawk–Dove when $0 < V < C$.

2.6.3 To see this, we need to turn the game into a dynamical system. Suppose that $x_1(t)$ is the proportion of Hawks in the population, and similarly $x_2(t)$ for Doves at an arbitrary point in time t. We wish to find what these quantities are after a small interval Δt. The simplest (most parsimonious, one might say) way of doing this is by putting

$$x_i(t + \Delta t) = x_i(t) + \Delta t x_i(t)(f_i(t) - \phi(t)) \tag{5}$$

for each $i = 1, 2$, where $f_i(t)$ is again the fitness of strategy i and $\phi(t)$ is a normalization term to be explicated shortly.

2.6.3.1 Why? Because, under an extremely simple construal of asexual Darwinian reproduction, the rate of increase or decrease in x_i should be dependent on three things: how long is the interval we are considering (Δt); how many i-strategists there already are, i.e. how many potential parents there are ($x_i(t)$); and how fit the i-strategy is, i.e. how successful those strategists are in producing offspring ($f_i(t) - \phi$). It is possible to interpret $f_i(t)$ as a **birth rate** and $\phi(t)$ as a **death rate**.

2.6.4 Rearranging the terms and dividing by Δt then leads to

$$\frac{x_i(t + \Delta t) - x_i(t)}{\Delta t} = x_i(t)(f_i(t) - \phi(t)). \tag{6}$$

If we let Δt tend to 0, we arrive at the differential equation

$$\dot{x}_i = x_i(f_i - \phi) \tag{7}$$

where I have dropped the explicit time reference for conciseness and $\dot{x}_i$ is Newton's notation for time derivatives. This is known as the **replicator equation**.

2.6.4.0.1   The "if we let Δt tend to 0" bit may seem (mathe)magical if you haven't done limits before. It boils down to how derivatives are defined, formally. I can't fit the details in here, but they can be found in any introductory treatment of real analysis.

2.6.4.0.2   And what is ϕ? It is defined to be the average fitness of the population,

$$\phi = \sum_i f_i x_i, \tag{8}$$

since, it can be shown, this ensures that the strategy densities sum to unity at any point of time under the replicator dynamics; always $\sum_i x_i = 1$.

2.6.4.1   We can now make some elementary observations. Firstly, as a differential equation, (7) gives the rate of change in the proportion of i-strategists in the population as a function of that proportion and the fitness of that strategy. Studying the behaviour of this equation then gives us a way to study the dynamics of the game itself. Secondly, notice that (7) can be positive, negative or zero depending on the relative magnitudes of f_i and ϕ. Roughly: if the fitness of i, f_i is greater than the average fitness ϕ, the proportion of i-strategists will increase; if it is less, that proportion will decrease. This makes sense.

2.6.5   It is now possible to find out what are the points where $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$ in Hawk–Dove and to study their stability, in order to see if there is an evolutionarily stable state (a stable polymorphism or a stable mixed strategy, depending on our interpretation). We can sort of convince ourselves that one such state always exists for this game (insert formulae, figures, and handwaving...); this is when $x_1 = V/C$ and $x_2 = (C - V)/C$.

2.7   Before moving on, it is important to review some basic assumptions that we've made here implicitly.

2.7.1   Populations are infinite. In order to write down an equation like (7), we must be able to talk of strategy densities, i.e. of the probability of encountering any given strategy. In a finite population there is always variance due to sampling error. Thus consider a finite vs. an infinite number of coin tosses. If the coin is fair, the probability of heads is $1/2$ for each individual toss. However, for any small number N of coin tosses, the average number of heads encountered may disagree with $1/2$ considerably. It is only in the limit $N \to \infty$ that this average converges to the expected $1/2$ (Law of Large Numbers). Similarly, in a two-strategy game there is some probability p of encountering one of the strategists, and we can keep track of which kind of strategists each of the players encounters during their lifetime. If N, the number of players is small, the empirical average may disagree with the expected one, just as in the coin toss example; but in the theoretical limit of an infinite population,

the agreement will be exact. (Another enlightening way to think about this is by noting that, in an infinite population, the probability of encountering any *particular* individual is zero, while in a finite population it is nonzero and will generally be the greater the smaller the population.) Finite populations thus introduce interesting stochastic effects, and their study is more complicated (more on this below).

2.7.2   We have also assumed that populations are randomly mixing. That is, each individual has an equal probability of coming into contact with each other individual.

2.7.3   We are focusing on **two-player games**. That is, each contest is assumed to involve two players only.

2.7.4   We are focusing on **symmetric games**. That is, the payoff issuing from playing a given strategy does not depend on the identity of the player.

2.7.5   We are focusing on **memoryless games**. The contestants have no memory of their or their opponents' past actions.

2.7.6   Relaxing one or more of these assumptions is very much an active topic of research in contemporary EGT and leads to dynamics of greater complexity.

2.8   For a second example, imagine a payoff matrix like this:

$$B = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}. \tag{9}$$

This is the simplest form of a **Rock–Paper–Scissors** game (RPS). Visual inspection of the matrix suggests that Rock beats Scissors, which beats Paper, which beats Rock, and it can be analytically shown (though not here) that replicator dynamics under the payoffs (9) is indeed cyclical about the fixed point $x^* = (x_1, x_2, x_3) = (1/3, 1/3, 1/3)$.

2.8.1   There's an important lesson here: even though x^* is an equilibrium for RPS, it is not asymptotically stable. In other words: a *static* polymorphism of Rocks, Papers and Scissors can be sustained in theory only. Any slight perturbation from the equilibrium will lead the system onto a periodic orbit where the relative abundances of the three strategies fluctuate in a cycle. It is a sort of a dynamic polymorphism.

2.8.2   There's also some fun real-world science here. It turns out that a Californian lizard, *Uta stansburiana*, in fact plays RPS. Their males come in three varieties with behavioural traits that are organized in the cyclical RPS way. Somewhat unromantically, we may view the female U. stansburiana as a finite resource which the males are vying for. The aggressive "ultradominant" male variant (also called Orange for his distinctive throat colouring) keeps a maximal number of wives and is good at suppressing the less aggressive and more diminutive "dominant" variant (Blue), which keeps a smaller number. However, he is liable to be fooled by the "sneaker" variety (Yellow), whose strategy is to resemble the females phenotypically and steal the partners of

the other variants. The sneaker is, on the other hand, not very successful in contests with the dominant, whose attention is not as widely divided as that of the ultradominant. So a cycle ensues: $O \to Y \to B \to O \to \dots$ Time series mapped from an actual U. stansburiana community show a period of about six years (Sinervo & Lively, 1996).

3   "... and it happened that they were so quiet not out of cunning, nor out of fear, but out of listening..." (Rilke.)

3.1   We now need a way of making sense of language change as a game. What are the strategies? What are the payoffs?

3.1.1   Thinking of this in an acquisition-based framework, we identify strategies with grammars.

3.1.1.1 And, for simplicity, we further identify grammars with the languages they generate.

3.1.1.2 Each possible strategy, then, is a subset $L_i \subseteq \Sigma^*$, Σ^* being the set of well-formed expressions in an alphabet Σ.

3.1.1.3 Conversely, each such set L_i is a strategy.

3.1.1.3.1 And, consistent with the symbolism above, we refer to the proportion of L_i strategists in our population with the symbol x_i.

3.1.2   Suppose now that each strategy i comes with a probability distribution P_i over Σ^* with support in L_i. For any $A \subseteq \Sigma^*$, we interpret $P_i(A)$ as the probability that an L_i speaker utters a sentence in the set A.

3.1.2.1 Then $\beta_{ij} = P_i(L_i \cap L_j)$ is the probability that an L_i speaker utters something that an L_j speaker can parse.

3.1.3   With Komarova, Niyogi & Nowak (2001), we can take $b_{ij} = \frac{1}{2}(\beta_{ij} + \beta_{ji})$ as the payoff in a situation where an L_i speaker meets an L_j speaker.

3.1.3.1 Does this mean that speakers who are better understood have greater reproductive success?

3.1.3.1.1 This is, indeed, the (dubious) interpretation given in Komarova, Niyogi & Nowak (2001).

3.1.3.1.2 But it is not necessary. Instead of this speaker-centred view, we may take a grammar-centred interpretation. Then each grammar will have a fitness in roughly the following sense: the probability of the grammar being passed on in processes of acquisition is the larger the more useful it is in terms of communication, *in relation to what the distribution of grammars in use in the population happens to be.*

3.1.3.1.2.1 This is not unreasonable, since intuitively children will have a tendency to acquire grammars which are useful for parsing the PLD they are exposed to.

3.1.3.2   There is a more general lesson here: a mathematical form like (7) can be interpreted in many ways.

3.1.3.2.1   The form itself does not care.

3.1.3.2.2   The interpretation is beneath the form.

3.1.3.3   So we can have a cultural-transmission as well as a biological-transmission interpretation of something like replicator dynamics. This is an important realization, for it safeguards us against rash accusations of "biologism", the putatively harmful extension of biological notions to domains non-biological.

3.1.3.3.1   When in reality the situation is something like the following: we *are* modelling cultural transmission, only clouding some (many) of its details, like the psychological details of language acquisition.

3.1.3.3.2   This is not unlike the original biological application of EGT which, experimental biologists will not tire of telling us, clouds many details of biological transmission and dynamics.

4   "Changing, it rests." (Heraclitus, fragment 84a.)

4.1   The replicator dynamics that equation (7) governs is rather simple – regardless of what particular game (what particular payoff matrix) we are considering.

4.1.1   In particular, the equation has no intrinsic mechanism for **mutation**, the sort of event when there is a "copying error" from parent to child.

4.1.2   But nature has this kind of thing.

4.2   If we wish to have an EGT approach to language change, a mutation mechanism seems a particularly important thing to have. Otherwise we'd have to assume that linguistic transmission is always faithful. If we think of change as driven by vagaries of acquisition, this would imply that children always converge on the grammar of their parents. But we know this is not the case.

4.3   It appears, then, necessary to generalize the simple replicator dynamics. An elegant way of doing this is provided by the **replicator–mutator equation** (for the most extensive treatment of this, see Mitchener, 2003).

4.4   Suppose there are n strategies (grammars). Then, according to the replicator–mutator equation, the first time derivative of the proportion of grammar i in the population is given by

$$\dot{x}_i = \sum_{j=1}^{n} q_{ji} f_j x_j - \phi x_i, \qquad (10)$$

where q_{ji} is to be interpreted as the probability that a j strategist gives rise to an i strategist (instead of another j strategist as faithful copying would imply).

4.4.0.1 We can put these q_{ji} into a matrix, a **mutation matrix**, so:

$$Q = [q_{ij}] = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{pmatrix}, \tag{11}$$

where the magnitude of the diagonal values q_{ii} is usually assumed to be much greater than then the magnitude of values off-diagonal (so, transmission is faithful in most cases). The matrix needs be row-stochastic.

4.4.1 So why does this generalize the replicator equation (7)? By rearranging the terms we can break (10) into two components, those of replication and mutation:

$$\begin{aligned} \dot{x}_i &= \sum_{j=1}^{n} q_{ji} f_j x_j - \phi x_i \\ &= \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ji} f_j x_j - q_{ii} f_i x_i - \phi x_i \\ &= \underbrace{\sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ji} f_j x_j}_{\text{mutation}} + \underbrace{x_i(q_{ii} f_i - \phi)}_{\text{replication}}. \end{aligned} \tag{12}$$

So putting $q_{ii} = 1$ and $q_{ji} = 0$ for $j \neq i$ (faithful transmission) restores the replicator equation.

4.5 Equation (10) is a system of n nonlinear and dependent differential equations, each of which has n^2 payoff parameters and n^2 mutation parameters. It is scarcely possible to solve it in the general case. Progress has been made, however, by limiting n to small values (e.g. $n = 3$) or by considering very simple, usually symmetric payoff and mutation matrices.

4.5.1 The most important result seems to be the following (see Komarova, Niyogi & Nowak, 2001). In a situation where grammars are symmetric, so that b_{ij} equals some constant b for all i, j, the nature of the mutation matrix determines whether the language community tends to a "coherent" or an "incoherent" state. More formally: if we define the mutation matrix by putting $q_{ii} = \mu$ with some constant μ for all i and $q_{ij} = (1 - \mu)/(n - 1)$ for $i \neq j$, then μ acts as a **bifurcation parameter** in the following sense:

4.5.1.1 There exist two critical values, μ_1 and μ_2, at which the dynamics of the language community bifurcates. With $\mu < \mu_1$, the community will be attracted toward a stable state in which each grammar is equinumerous (has frequency $1/n$). With $\mu_1 < \mu < \mu_2$ another fixed point emerges, at which one particular grammar dominates in the sense that the frequencies of other grammars will be negligible. With $\mu_2 < \mu$ the symmetric fixed point loses stability, so at such high values of μ the community will always be driven towards coherence.

4.5.1.1.1 This is perhaps not so surprising. What it says is, in effect, that if children are faithful learners, language communities are coherent (employ mostly one and the same grammar); and, if they're not, then the community will not be coherent. However, under this seemingly trivial observation there lie many interesting features to the dynamics that are of interest to at least mathematicians: the dynamics can display limit cycles, so-called Hopf bifurcations and even deterministic chaos (Mitchener & Nowak, 2004). Particularly interesting from a language change point of view is the way in which the system may be perturbed from one asymmetric fixpoint to another. How to go from Old English to Modern English? According to this model, the only way is to make the journey via a less coherent language state during a period where μ is below μ_2 and thus neither Old English nor Modern English is attracting.

4.6 In a very different sort of application, Quinley & Mühlenbernd (2012) set up a simple game-theoretical scenario to explain how meanings are attached to new lexical items in a situation of language contact. The basic idea is simple: agents engage in communication situations called "signaling games" where different strategies constitute different mappings between lexical items and meanings. An agent is assumed to receive a positive payoff if he, as a listener, construes the meaning the speaker intended, and a zero payoff otherwise. (Again, this does not mean that successful communication increases an agent's reproductive success in the population, but that the density of a successful strategy will tend to increase across the population.) Based on computer simulations, the authors suggest that for the context-sensitive meanings of pairs such as *swine–pork* and *cow–beef* to develop in a systematic manner in a situation of language contact certain prestige and/or social network effects must be in operation.

5 "...indeed the gritty details are always messier than the abstractions on the linguist's desk." (Árnason, 1986, 21.)

5.1 And, I would say, this holds *a fortiori* of the abstractions on the evolutionary game theorist's desk.

5.2 To paraphrase an instructive analogy which Mitchener (2004) has previously employed in a similar context: this is like trying to reproduce Mona Lisa in a line drawing, getting the abstract essence of her smile right but all the details, gritty or not, wrong – or at least only approximatively right.

5.3 That is, we believe that simple formulations like those considered above really do reveal some underlying regularities that dynamics of populations, be they animal or human, observe. However, we acknowledge that reality is much more complex than that.

5.3.1 Real populations are not infinite; individuals do not mix randomly; individuals are located in specific points in space, they may be involved in social networks, they may learn and update their strategies over time; the transmission of a strategy from one generation to the next may be very much more complicated than the simple asexual copying that classical games in EGT assume.

5.3.2 Interestingly, and importantly, many classical EGT results fail to hold when

some of the classical idealizations are dropped. For example:

5.3.2.1   Nowak & May (1992) show that if individuals are constrained to mix on a spatial grid, some classically unstable strategy mixes may become stable.

5.3.2.2   Galla (2009) considers a two-player setting with mixed strategies and "batch learning": each player is assumed to update the probabilities with which they play the various pure strategies at certain finite mileposts, keeping track of which strategies have worked best in the past. There is also a memory loss parameter which controls how far into history the players can see. This leads to dynamics that differ from those of classical setups. For instance, in Rock–Paper–Scissors the central equilibrium undergoes a bifurcation when the memory loss parameter crosses a critical value: with low memory loss, the fixed point is repelling and with high memory loss attracting, modulo some noise-sustained oscillations about the fixed point. Thus, cyclicity becomes either an outward or an inward spiral.

5.3.2.2.1   This sort of framework may hold the seeds of an interesting usage-based approach to modelling language change: it allows for mixed strategies, which in this case would be something like probabilistic grammars, as well as grammar updates during the lifetime of an individual. To my knowledge, no-one has explored this possibility so far, although the Utterance Selection Model of Baxter, Blythe, Croft & McKane (2006) comes close in some respects.

5.4   Why focus on simple formulations then?

5.4.1   Historical reasons: classical EGT dates from a time when only the simplest of systems could be simulated computationally.

5.4.1.1   And analytical (pencil-and-paper) progress very soon becomes impossible as the realism of the models is scaled up. For the mathematically minded: observe that even an equation as simple as (7) is nonlinear, since the fitness terms actually contain x_1 and x_2 as factors. So, in general it is not possible to solve these things analytically. Stability can be investigated using e.g. Lyapunov analysis, but this is not easy.

5.4.2   The computational power available to us now makes investigation of very much more complicated systems and games possible, by simulating them. However, it is not clear that merely throwing in more and more complexity into a computer simulation will actually increase our understanding.

5.4.2.1   A faithful reproduction of Mona Lisa is a copy which, certainly, does contain the essence – but it does not tell you where or what that essence is. So it appears there are epistemological reasons for continuing to work with simple formulations.

5.5   That said, in my opinion some of the most interesting contemporary work in EGT has to do with relaxing some of the idealizing assumptions made in classical EGT and pushing the boundaries of the framework, using e.g. methods from non-equilibrium statistical physics and relaxing the hold that neo-Darwinism has on EGT. It is an instance of moving, in tiny and careful steps,

from what is simple and pure to something a bit more complex and hopefully more realistic. It is debatable whether work of this kind still represents EGT in its true spirit – if, for instance, you drop the assumption of fitness-driven evolution and consider **neutral evolution** driven by stochastic demographic effects, are you still doing EGT? What's your game? Luckily the question itself has little cognitive content, for

6   "There is no idea, however ancient and absurd, that is not capable of improving our knowledge. The whole history of thought is absorbed into science and is used for improving every single theory." (Feyerabend, 1975/1993, 33.)

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